MTH251	Sample Final Exam	Fall 2018

Name:	Date:

Show all work if full or partial credit is desired. You may use a graphing calculator or Desmos on the second part. If Desmos is used, then the device must be in airplane-mode (i.e., no wifi, cellular, or Bluetooth connections). No notes, books or websites allowed.

Part 1 No Calculator

1. $\lim_{x \to -3} \left(x^2 - 2x + 5\right) =$

2. $\lim_{x \to 0+} \frac{1}{x} =$

3. $\lim_{x \to \infty} \frac{1}{x} =$

4. $\lim_{x \to 5} \frac{x^2 - 25}{x - 5} =$

5. $f(x) = 7$, so $f'(x) =$

6. $g(x) = x$, so $\frac{d}{dx}[g(x)] =$

7. $q(t) = t^3$, so $\frac{d}{dt}[q(t)] =$

8. $\frac{d}{d\theta}[\cos(\theta)] =$

9. Use the limit definition of the derivative to find the slope of the tangent line to the curve $r(x) = \frac{2}{x}$ at $x = -3$.

Part 2 Calculator Allowed

Find the following derivatives.

10. $\frac{d}{dx}\left[2x^2 - 3x + 5\right] =$

11. $\frac{d}{dx}\left[(x-3)^2\right] =$

12. $\frac{d}{dx}\left[3\sqrt{x}\right] =$

13. $\frac{d}{dx}\left[\left(x^2 + 3x + 6\right)\left(x^2 - 3x \pm 5\right)\right] =$

14. $\frac{d}{dx}\left[\frac{x^2+3}{x+3}\right] =$

15. $\frac{d}{dx}[\cos(x)\sin(x)] =$

16. $\frac{d}{dx}\left[\sqrt{4-x^2}\right] =$

17. $\frac{d}{dx}\left[\{\sin(\pi x + 1)\}^3\right] =$

18. $\frac{d}{dx}\left[\frac{e^x - e^{-\times}}{2}\right] =$

19. $\frac{d}{dx}[\arccos(x)] =$

20. $\frac{d}{dx}[\ln(5x)] =$

21. $\frac{d}{dx}[\ln(\cos(x))] =$

22. $\frac{d}{dx}[2 \cdot 3^x] =$

Fun with Derivatives

23. Calculate the slope of the tangent line (rate of change) to $f(x) = \dfrac{3}{x-1}$ at $x = -1$ by finding the derivative function.

   a. $\dfrac{d}{dx}[f(x)] =$

   b. $f'(-1) =$

   c. The equation of the tangent line is ___________________

   d. Sketch a graph of $f(x)$ on the interval $[-3, 1]$ along with the tangent line when $x = -1$.
   Be sure to label and scale the axes, and use a ruler if necessary.

24. A ball is dropped from a height of 25 meters. Its height above ground (in meters) t seconds later is given by $h(t) = -4.9t^2 + 25$.

   a. At what time does the ball hit the ground?

   b. What is the instantaneous velocity of the ball when it hits the ground?

   c. What is the average velocity during its fall?

25. Find the slope of the tangent line to the curve $x^3 - 9xy + y^3 = 0$ at the point $(2, 4)$.

26. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ if $y = (1+x)^{\frac{1}{x}}$.

27. If $P(t) = 1000e^{0.3t}$ describes a mosquito population after t days, what is the rate of change of the population after 4 days?

Part 3: Chapter 4

28. Consider the volume of a sphere, $V = \frac{4}{3}\pi r^3$. Suppose that you fill the balloon with air at a constant rate, $100\frac{\text{cm}^3}{\text{s}}$.
    a. At what rate does the radius increase when the radius is 2cm?

    b. At what rate does the radius increase when the radius is 4cm?

29. The diameter of a spherical ball bearing was measured to be 5mm with a possible error of 0.05mm. Use linear approximation (aka differentials) to **estimate** the maximum error in the volume of the ball bearing.

30. Consider the function $f(x) = x^3 - 7x^2 + 8x + 1$
    a. Find $\frac{dy}{dx}$.

    b. Find any critical values.

    c. Use the first derivative to identify any intervals where the function is increasing.

    d. Use the first derivative to identify any intervals where the function is decreasing.

    e. Find $\frac{d^2y}{dx^2}$.

    f. Use the second derivative and the critical values to find any extreme values.

    g. Use the second derivative to identify intervals where the function is concave up.

    h. Use the second derivative to identify intervals where the function is concave down.

    i. Identify any inflection points.

31. Consider the function $f(x) = \frac{x^2 + 7x + 10}{x + 1}$
    a. Find $\frac{dy}{dx}$.

    b. Find any critical values.

    c. Use the first derivative to identify any intervals where the function is increasing.

    d. Use the first derivative to identify any intervals where the function is decreasing.

    e. Find $\frac{d^2y}{dx^2}$.

    f. Use the second derivative and the critical values to find any extreme values.

    g. Use the second derivative to identify intervals where the function is concave up.

    h. Use the second derivative to identify intervals where the function is concave down.

    i. Identify any asymptotes.

32. Consider an open-top box with a square base and a volume of $216 \ \text{in.}^3$. Suppose the cost of the material for the base is 20¢/in.^2 and the cost of the material for the sides is 30¢/in.^2 and we are trying to minimize the cost of this box. Write the cost as a function of the side lengths of the base. (Let x be the side length of the base and y be the height of the box.)

33. Use L'Hôpital's rule to find $\lim_{x \to \infty} \frac{x^2}{e^x}$.

34. Use Newton's method to approximate a root of $f(x) = x^3 - 3x + 1$ on the interval $[1, 2]$. Let $x_0 = 2$ and find x_1, x_2, x_3, x_4, and x_5.