MTH251	Sample Exam 1	Fall 2018

Name:	Date:

Show all work if full or partial credit is desired. You may use a graphing calculator or Desmos. If Desmos is used, then the device must be in airplane-mode (i.e., no wifi, cellular, or Bluetooth connections). No notes, books or websites allowed.

Part 1 No Calculator

1. $\lim_{x \to 2} 3 =$

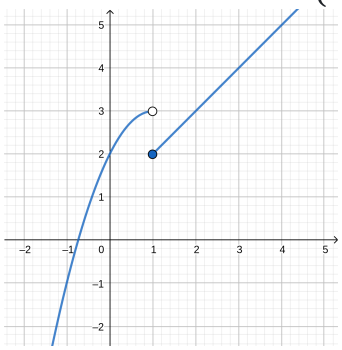
2. $\lim_{x \to 5} x =$

3. $\lim_{x \to -2} \left(x^2 + 3x - 4\right) =$

4. $\lim_{x \to 0+} \dfrac{1}{x} =$

5. $\lim_{x \to 7} \dfrac{x^2 - 49}{x - 7} =$

6. For what values of x is $f(x) = \begin{cases} 3 - (x-1)^2 & x < 1 \\ x + 1 & x \geq 1 \end{cases}$ continuous?



7. Use the limit definition of the derivative to find the slope of the tangent line to the curve $f(x) = x^2$ at $x = -2$.
   a. $f(-2+h) =$

   b. $f(-2+h) - f(4) =$

   c. $\dfrac{f(-2+h) - f(4)}{h} =$

   d. $\lim_{h \to 0} \dfrac{f(-2+h) - f(-2)}{h} =$

   e. So, $f'(-2) =$

Part 2 Calculator Allowed

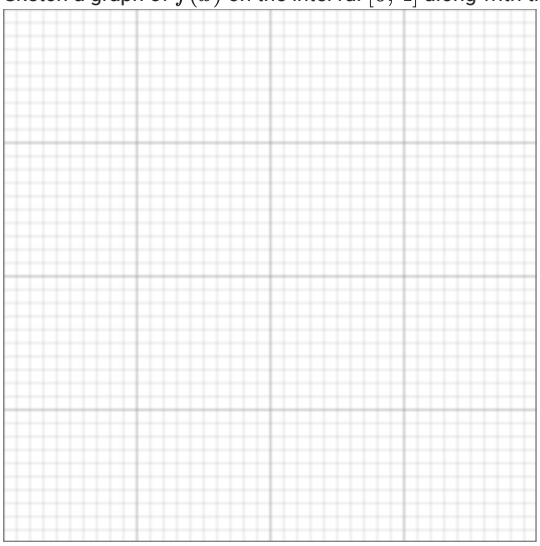
Secants and Tangents

1. Estimate the slope of the tangent line (rate of change) to $f(x) = \dfrac{1}{x}$ at $x = 2$ by finding the slopes of the secant lines through the points:
   a. $x = 2.1$ and 1.9

   b. $x = 2.01$ and 1.99

   c. Use the slopes of the secants to estimate the slope of the tangent line accurate to 2 decimal places.

   d. Sketch a graph of $f(x)$ on the interval $[0, 4]$ along with the tangent line when $x = 2$.



Evaluate the following limits.

2. $\lim_{x \to 7} \dfrac{x^2 - 49}{x - 7} =$

3. $\lim_{x \to -3} \left(2x^2 + 4x - 5\right) =$

4. $\lim_{x \to 2^-} \dfrac{1}{x - 2} =$

5. $\lim_{\theta \to 0} \dfrac{\sin(4\theta)}{4\theta} =$

6. $\lim_{x \to 0} (1 + 2x)^{\frac{1}{x}} =$

7. $\lim_{x \to 0} \dfrac{x - 9}{\sqrt{x} - 3} =$

8. $\lim_{x \to 0} \dfrac{\sqrt{3x + 1} - 1}{2x} =$

9. $\lim_{x \to 0} \left(\dfrac{3}{x^2 - x} + \dfrac{1}{x}\right) =$

10. If $f(x) = \begin{cases} (x + 3)^2 - 1 & x < -1 \\ 2x - 1 & x \geq -1 \end{cases}$

    a. $\lim_{x \to -1^-} f(x) =$

    b. $\lim_{x \to -1^+} f(x) =$

    c. $\lim_{x \to -1} f(x) =$

Continuity

11. Determine whether $r(x) = \dfrac{x^2 - 49}{x - 7}$ is continuous at $x = 7$. If it is continuous, explain why. Otherwise, explain why it is discontinuous.

12. Determine whether $p(x) = x^2 - 14x + 49$ is continuous at $x = 7$. If it is continuous, explain why. Otherwise, explain why it is discontinuous.

13. Consider the function $f(x) = \dfrac{1}{x}$ on the interval $-1 \leq x \leq 1$.
    $f(-1) = -1$ and $f(1) = 1$, so, is there a value of c in $[-1, 1]$ such that $f(c) = 0$? Why or why not?

14. For what values of x is $g(x) = \dfrac{x - 5}{x + 2}$ continuous?

The Precise Definition of a Limit

15. State the Precise Definition of a Limit.

16. If $f(x) = 2x - 1, a = -1$, and $\varepsilon = 0.01$, then find an appropriate value of δ.

17. If $f(x) = \frac{1}{2}x + 1$ and $a = 4$, then find δ in terms of ε.

The Derivative at a Point

18. Use the limit definition of the derivative to find the slope of the tangent line to the curve $f(x) = 5x^2$ at $x = 4$.
    Evaluate each of the following and state your answers in simplest form:
    a. $f(4+h) =$

    b. $f(4+h) - f(4) =$

    c. $\dfrac{f(4+h) - f(4)}{h} =$

    d. $\lim\limits_{h \to 0} \dfrac{f(4+h) - f(4)}{h} =$

    e. So, $f'(4) =$

19. Use the limit definition of the derivative to find the slope of the tangent line to the curve $f(x) = 4x^2$ at $x = 1$.

20. $f(x) = \sqrt{25 - x}$. Use the limit definition of the derivative to compute $f'(9)$.

The Derivative Function

21. Use the limit definition of the derivative to find the derivative function when $f(x) = \frac{5}{x}$.

    Evaluate each of the following and state your answers in simplest form:

    a. $f(x+h) =$

    b. $f(x+h) - f(x) =$

    c. $\frac{f(x+h) - f(x)}{h} =$

    d. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$

    e. So, $f'(x) =$

22. Use the limit definition of the derivative to find the derivative function when $f(x) = 3x^2$.

23. $f(x) = \sqrt{25 - x}$. Use the limit definition of the derivative to find the derivative function $f'(x)$.

24. $f(x) = \frac{x}{1 - x^2}$. Use the limit definition of the derivative to find the derivative function $f'(x)$.

25. $f(x) = \frac{1}{x}$. Use the limit definition of the derivative to find the second derivative function $f''(x)$.