Patterns

1. $f'(x) = \dfrac{d}{dx}[x] = \lim_{h \to 0} \dfrac{(x+h) - x}{h} = 1$

2. $f'(x) = \dfrac{d}{dx}\left[x^2\right] = \lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} (2x+h) = 2x$

3. $f'(x) = \dfrac{d}{dx}\left[x^3\right] = \lim_{b \to x} \dfrac{x^3 - b^3}{x - b} = \lim_{b \to x} \left(x^2 + bx + b^2\right) = 3x^2$

4. $f'(x) = \dfrac{d}{dx}\left[\dfrac{1}{x}\right] = \lim_{b \to x} \dfrac{\frac{1}{x} - \frac{1}{b}}{x - b} = \lim_{b \to x} \dfrac{b - x}{xb} \cdot \dfrac{1}{x - b} = \lim_{b \to x} \dfrac{-1}{xb} = \dfrac{-1}{x^2} = -x^{-2}$

5. $f'(x) = \dfrac{d}{dx}\left[\sqrt{x}\right] = \lim_{b \to x} \dfrac{\sqrt{x} - \sqrt{b}}{x - b} = \lim_{b \to x} \dfrac{1}{\sqrt{x} + \sqrt{b}} = \dfrac{1}{2\sqrt{x}} = \dfrac{1}{2}x^{-\frac{1}{2}}$

6. What is the pattern?

General Rules for the derivative:

The Easy Ones

1. $\frac{d}{dx}[a \cdot f(x)] = a \cdot \frac{d}{dx}[f(x)]$

2. $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)] = f'(x) + g'(x)$

The Harder Ones

1. $\frac{d}{dx}[f(x) \cdot g(x)] = \ ?$

2. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \ ?$

$$\frac{d}{dx}[f(x) \cdot g(x)] = \lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x)}{h}$$

The trick is to add and subtract $f(x+h) \cdot g(x)$ in the numerator.

$$\lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x)}{h} = \lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x+h) \cdot g(x) + f(x+h) \cdot g(x) - f(x) \cdot g(x)}{h}$$

Now, factor in groups.

$$\lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x+h) \cdot g(x) + f(x+h) \cdot g(x) - f(x) \cdot g(x)}{h} = \frac{f(x+h)[g(x+h) - g(x)] + g(x)[f(x+h) - f}{h}$$

Now separate these two groups.

$$\lim_{h \to 0} \frac{f(x+h)[g(x+h) - g(x)] + g(x)[f(x+h) - f(x)]}{h} = \lim_{h \to 0} \frac{f(x+h)[g(x+h) - g(x)]}{h} + \lim_{h \to 0} \frac{g(x)[f(x+h) - f(x)]}{h}$$

We can evaluate these limits based on our limit rules.

$f(x) \cdot g'(x) + g(x) \cdot f'(x)$